Introducing Temporal Description Logics

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1 Introduction

Description Logics are formalisms designed for a logical reconstruction of representation tools such as *frames*, *Object-Oriented* and *semantic* data models, *semantic networks*, *type systems*, and *feature logics*. Nowadays, description logics are also considered the most important unifying formalism for the many object-centred representation languages used in areas other than Knowledge Representation. Important characteristics of Description Logics are high expressivity together with decidability, which guarantee that reasoning algorithms always terminate with the correct answers.

This short paper will introduce temporal extensions of Description Logics, through the detailed analysis of a case study, involving the combination of a rather expressive Description Logic with the basic tense modal logic over a linear, unbounded, and discrete temporal structure. $\mathcal{ALCQIT}$ is the temporal description logic considered as the case study. This language is obtained by combining a standard tense logic and the non-temporal description logic $\mathcal{ALCQI}$ with axioms. We first introduce the non-temporal part of the language, and then we will present its combination with the tense logic.

At the end of the paper we will briefly report how other approaches in extending Description Logics with time relate to the case study.

2. Description Logics

In this section we give a brief introduction to the $\mathcal{ALCQI}$ description logic, which will serve as the basic representation language for the non-temporal information. With respect to the formal apparatus, we will strictly follow the standard concept language formalism whose extensions have been summarised in [5, 3]. In this perspective, Description Logics are considered as a *structured* fragment of predicate logic. $\mathcal{ALC}$ is the minimal description language including full negation and disjunction—i.e., propositional calculus, and it is a notational variant of the propositional multi-modal logic $\boldsymbol{K_{(m)}}$ [9] (see next section).

C, D	$\rightarrow$	$A \mid$	A	(primitive conc.)
		$\top \mid$	`top`	(top)
		$\bot \mid$	`bottom`	(bottom)
		$\neg C \mid$	(`not` C)	(complement)
		$C \sqcap D \mid$	(`and` C D $\ldots$)	(conjunction)
		$C \sqcup D \mid$	(`or` C D $\ldots$)	(disjunction)
		$\forall R.C \mid$	(`all` R C)	(univ. quantifier)
		$\exists R.C \mid$	(`some` R C)	(exist. quantifier)
		$f\uparrow \mid$	(`undefined` f)	(undefinedness)
		$f : C \mid$	(`in` f C)	(selection)
		$\geq n\ R.C \mid$	(`atleast` n R C)	(min cardinality)
		$\leq n\ R.C$	(`atmost` n R C)	(max cardinality)
R	$\rightarrow$	$P \mid$	P	(primitive role)
		$f \mid$	f	(primitive feature)
		R^{-1}	(`inverse` R)	(inverse role)

Figure 1. Syntax rules for $\mathcal{ALCQI}$

The basic types of a concept language are *concepts*, *roles*, and *features*. A concept is a description gathering the common properties among a collection of individuals; from a logical point of view it is a unary predicate ranging over the domain of individuals. Inter-relationships between these individuals are represented either by means of roles (which are interpreted as binary relations over the domain of individuals) or by means of features (which are interpreted as partial functions over the domain of individuals). In the following, we will consider the Description Logic $\mathcal{ALCQI}$, extending $\mathcal{ALC}$ with qualified cardinality restrictions and inverse roles.

According to the syntax rules of Figure 1, $\mathcal{ALCQI}$ *concepts* (denoted by the letters C and D) are built out of *primitive concepts* (denoted by the letter A), *roles* (denoted by the letter R), and *primitive features* (denoted by the letter

$$\begin{aligned}
\top^{\mathcal{I}} &= \Delta^{\mathcal{I}} \\
\bot^{\mathcal{I}} &= \emptyset \\
(\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
(C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(\forall R.\, C)^{\mathcal{I}} &= \{i \in \Delta^{\mathcal{I}} \mid \forall j.\, R^{\mathcal{I}}(i,j) \Rightarrow C^{\mathcal{I}}(j)\} \\
(\exists R.\, C)^{\mathcal{I}} &= \{i \in \Delta^{\mathcal{I}} \mid \exists j.\, R^{\mathcal{I}}(i,j) \wedge C^{\mathcal{I}}(j)\} \\
(f\uparrow)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus \operatorname{dom} f^{\mathcal{I}} \\
(f : C)^{\mathcal{I}} &= \{i \in \operatorname{dom} f^{\mathcal{I}} \mid C^{\mathcal{I}}(f^{\mathcal{I}}(i))\} \\
(\geq n\, R.\, C)^{\mathcal{I}} &= \{i \in \Delta^{\mathcal{I}} \mid \sharp\{j \in \Delta^{\mathcal{I}} \mid R^{\mathcal{I}}(i,j) \wedge C^{\mathcal{I}}(j)\} \geq n\} \\
(\leq n\, R.\, C)^{\mathcal{I}} &= \{i \in \Delta^{\mathcal{I}} \mid \sharp\{j \in \Delta^{\mathcal{I}} \mid R^{\mathcal{I}}(i,j) \wedge C^{\mathcal{I}}(j)\} \leq n\} \\
(R^{-1})^{\mathcal{I}} &= \{(i,j) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid R^{\mathcal{I}}(j,i)\}
\end{aligned}$$

Figure 2. Extensional semantics of $\mathcal{ALCQI}$

$$\begin{aligned}
\top^{\mathcal{I}} &\sim \mathsf{true} \\
\bot^{\mathcal{I}} &\sim \mathsf{false} \\
(\neg C)^{\mathcal{I}} &\sim \neg F_C(\gamma) \\
(C \sqcap D)^{\mathcal{I}} &\sim F_C(\gamma) \wedge F_D(\gamma) \\
(C \sqcup D)^{\mathcal{I}} &\sim F_C(\gamma) \vee F_D(\gamma) \\
(\exists R.\, C)^{\mathcal{I}} &\sim \exists x.\, F_R(\gamma, x) \wedge F_C(x) \\
(\forall R.\, C)^{\mathcal{I}} &\sim \forall x.\, F_R(\gamma, x) \Rightarrow F_C(x) \\
(f\uparrow)^{\mathcal{I}} &\sim \neg \exists x.\, f(\gamma, x) \\
(f : C)^{\mathcal{I}} &\sim \exists x.\, f(\gamma, x) \wedge F_C(x) \\
(\geq n\, R.\, C)^{\mathcal{I}} &\sim \exists^{\geq n} x.\, F_R(\gamma, x) \wedge F_C(x) \\
(\leq n\, R.\, C)^{\mathcal{I}} &\sim \exists^{\leq n} x.\, F_R(\gamma, x) \wedge F_C(x) \\
(R^{-1})^{\mathcal{I}} &\sim F_R(\beta, \alpha)
\end{aligned}$$

Figure 3. FOL semantics of $\mathcal{ALCQI}$

f); roles are built out of *primitive roles* (denoted by the letter P) and *primitive features*. The top part of Figure 1 defines the $\mathcal{ALC}$ sublanguage. Please also note that features are introduced as shortcuts; in fact, they can be expressed by means of axioms using cardinality restrictions.

Let us now consider the formal semantics of $\mathcal{ALCQI}$. We define the *meaning* of concepts as sets of individuals—as for unary predicates—and the meaning of roles as sets of pairs of individuals—as for binary predicates. Formally, an *interpretation* is a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consisting of a set $\Delta^{\mathcal{I}}$ of individuals (the *domain* of $\mathcal{I}$) and a function $\cdot^{\mathcal{I}}$ (the *interpretation function* of $\mathcal{I}$) mapping every concept to a subset of $\Delta^{\mathcal{I}}$, every role to a subset of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, and every feature to a partial function from $\Delta^{\mathcal{I}}$ to $\Delta^{\mathcal{I}}$, such that the equations in Figure 2 are satisfied. The semantics of the language can also be given by stating equivalences among expressions of the language and First Order Logic formulae. An atomic concept A, an atomic role P, and an atomic feature f, are mapped respectively to the open formulæ$A(\gamma)$, $P(\alpha, \beta)$, and $f(\alpha, \beta)$ – with f a functional relation, also written $f(\alpha) = \beta$. Figure 3 gives the transformational semantics of $\mathcal{ALCQI}$ expressions in terms of equivalent FOL well-formed formulæ. A concept C and a role R correspond to the FOL open formulae $F_C(\gamma)$ and $F_R(\alpha, \beta)$ respectively. It is worth noting that, using the standard model-theoretic semantics, the extensional semantics of Figure 2 can be derived from the transformational semantics of Figure 3.

For example, we can consider the concept of HAPPY FATHERS, defined using the primitive concepts `Man, Doctor, Rich, Famous` and the roles `CHILD, FRIEND`. The concept HAPPY FATHERS can be expressed in $\mathcal{ALCQI}$ as

$$\begin{aligned}
&\mathtt{Man} \sqcap (\exists \mathtt{CHILD}.\top) \sqcap \\
&\ \forall \mathtt{CHILD}.(\mathtt{Doctor} \sqcap \exists \mathtt{FRIEND}.(\mathtt{Rich} \sqcup \mathtt{Famous})),
\end{aligned}$$

i.e., those men having some child and all of whose children are doctors having some friend who is rich or famous.

A *knowledge base*, in this context, is a finite set Σ of *terminological axioms*; it can also be called a *terminology* or TBox. For a concept name A, and (possibly complex) concepts C, D, terminological axioms are of the form $A \doteq C$ (concept definition), $A \sqsubseteq C$ (primitive concept definition), $C \sqsubseteq D$ (general inclusion statement). An interpretation $\mathcal{I}$ satisfies $C \sqsubseteq D$ if and only if the interpretation of C is included in the interpretation of D, i.e., $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$. It is clear that the last kind of axiom is a generalisation of the first two: concept definitions of the type $A \doteq C$ – where A is an atomic concept – can be reduced to the pair of axioms $(A \sqsubseteq C)$ and $(C \sqsubseteq A)$. Another class of terminological axioms – pertaining to roles R, S – are of the form $R \sqsubseteq S$. Again, an interpretation $\mathcal{I}$ satisfies $R \sqsubseteq S$ if and only if the interpretation of R – which is now a set of *pairs* of individuals – is included in the interpretation of S, i.e., $R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$. An interpretation $\mathcal{I}$ is a *model* of a knowledge base Σ iff every terminological axiom of Σ is satisfied by $\mathcal{I}$. If Σ has a model, then it is *satisfiable*; thus, checking for KB satisfiability is deciding whether there is at least one model for the knowledge base. Σ *logically implies* an axiom α (written $\Sigma \models \alpha$) if α is satisfied by every model of Σ. We say that a concept C is *subsumed* by a concept D in a knowledge base Σ (written $\Sigma \models C \sqsubseteq D$) if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for every model $\mathcal{I}$ of Σ. For example, the concept

$$\mathtt{Person} \sqcap (\exists \mathtt{CHILD}.\mathtt{Person})$$

denoting the class of PARENTS—i.e., the persons having at least a child which is a person—subsumes the concept

$$\begin{aligned}
&\mathtt{Man} \sqcap (\exists \mathtt{CHILD}.\top) \sqcap \\
&\ \forall \mathtt{CHILD}.(\mathtt{Doctor} \sqcap \exists \mathtt{FRIEND}.(\mathtt{Rich} \sqcup \mathtt{Famous}))
\end{aligned}$$

denoting the class of HAPPY FATHERS – with respect to the following knowledge base Σ:

$$\begin{aligned}
&\mathtt{Doctor} \doteq \mathtt{Person} \sqcap \exists \mathtt{DEGREE}.\mathtt{Phd}, \\
&\mathtt{Man} \doteq \mathtt{Person} \sqcap \mathtt{sex} : \mathtt{Male},
\end{aligned}$$

i.e., every happy father is also a person having at least one child, given the background knowledge that men are male persons, and that doctors are persons.

A concept C is satisfiable, given a knowledge base Σ, if there is at least one model $\mathcal{I}$ of Σ such that $C^{\mathcal{I}} \neq \emptyset$, i.e. $\Sigma \not\models C \equiv \bot$. For example, the concept

$$(\exists \mathtt{CHILD.Man}) \sqcap (\forall \mathtt{CHILD.}(\mathtt{sex} : \neg \mathtt{Male}))$$

is unsatisfiable with respect to the above knowledge base Σ. In fact, an individual whose children are not male cannot have a child being a man.

Concept subsumption can be reduced to concept satisfiability since C is subsumed by D in Σ if and only if $(C \sqcap \neg D)$ is unsatisfiable in Σ.

Reasoning in $\mathcal{ALCQI}$ (i.e., deciding knowledge base and concept satisfiability, deciding concept subsumption, and deciding logical implication) is decidable, and it has been proven to be an EXPTIME-complete problem [3]. Reasoning in the special case of $\mathcal{ALC}$ with empty knowledge bases Σ is PSPACE-complete [5].

3. Correspondence with Modal Logics

Schild in [9] proved the correspondence between $\mathcal{ALC}$ and the propositional normal multi-modal logic $\boldsymbol{K_{(m)}}$. $\boldsymbol{K_{(m)}}$ is the simplest multi-modal logic interpreted over Kripke structures: there are no restrictions on the accessibility relations. Informally, a concept corresponds to a propositional formula, and it is interpreted as the set of possible worlds over which the formula holds. The existential and universal quantifiers correspond to the possibility and necessity operators over different accessibility relations: $\Box_r C$ is interpreted as the set of all the possible worlds such that in every r-accessible world C holds; $\Diamond_r C$ is interpreted as the set of all the possible worlds such that in some r-accessible world C holds. Thus, roles are interpreted as the accessibility relations between worlds. A knowledge base Σ corresponds to constraints on the Kripke structures, by stating what are the necessary relations between worlds, and what are the formulas necessarily holding in some world. Thus, we can speak of satisfiability of a formula ϕ of $K_{(m)}$ *with respect to* a set of world constraints Σ.

Starting from the work of Schild, the work presented in [3] defines a very expressive modal logic – $\mathcal{ALCQI}_{reg}$ – which extends the expressivity of *converse-PDL*, i.e. propositional dynamic modal logic with the converse operator, with cardinality restrictions (called in the modal logics community *graded modalities*). They have proven the decidability of satisfiability in $\mathcal{ALCQI}_{reg}$ and its correspondence with a very expressive description logic, which includes $\mathcal{ALC}$, functional and general cardinality restrictions, inverse roles, and *regular expressions* over roles. Reasoning in $\mathcal{ALCQI}_{reg}$ is decidable, and it has been proven to be an EXPTIME-complete problem [3].

4. Adding Tense Logic

The tense-logical extension of the description logic $\mathcal{ALCQI}$ is able to describe the time-varying aspect of concepts. Let $\mathcal{ALCQIT}$ be the extension of $\mathcal{ALCQI}$ with the temporal operators $\mathcal{U}$ (Until) and $\mathcal{S}$ (Since). We add the following rule to the syntax presented in figure 1:

$$\begin{array}{lllll} C, D & \rightarrow & C\mathcal{U}D \mid & (C \ \mathtt{until}\ D) & \text{(until)} \\ & & C\mathcal{S}D \mid & (C \ \mathtt{since}\ D) & \text{(since)} \\ & & \Diamond^{+}C \mid & (\mathtt{somefut}\ C) & \text{(future existential)} \\ & & \Diamond^{-}C \mid & (\mathtt{somepast}\ C) & \text{(past existential)} \\ & & \Box^{+}C \mid & (\mathtt{allfut}\ C) & \text{(future universal)} \\ & & \Box^{-}C & (\mathtt{allpast}\ C) & \text{(past universal)} \end{array}$$

The tense operators $\Diamond^{+}$ (sometime in the future), $\Diamond^{-}$ (sometime in the past), $\Box^{+}$ (always in the future), and $\Box^{-}$ (always in the past) are derived operators: $\Diamond^{+}, \Diamond^{-}$ are defined as $\Diamond^{+}C \doteq \top\mathcal{U}C$, $\Diamond^{-}C \doteq \top\mathcal{S}C$, while $\Box^{+}, \Box^{-}$ are their duals.

The $\mathcal{ALCQIT}$ semantics naturally extends with time the non-temporal semantics presented in the previous section. A temporal structure $\mathcal{T} = (\mathcal{P}, <)$ is assumed, where $\mathcal{P}$ is a set of time points and $<$ is a strict linear order on $\mathcal{P}$. An $\mathcal{ALCQIT}$ *temporal interpretation* over $\mathcal{T}$ is a pair $\mathcal{M} \doteq \langle \mathcal{T}, \mathcal{I} \rangle$, where $\mathcal{I}$ is a function associating to each $t \in \mathcal{P}$ a standard non-temporal $\mathcal{ALCQI}$ interpretation, $\mathcal{I}(t) \doteq \langle \Delta^{\mathcal{I}}, R_0^{\mathcal{I}(t)}, \ldots, C_0^{\mathcal{I}(t)}, \ldots \rangle$, such that it satisfies the equations in figure 2 plus the following:

$$\begin{array}{lll} (C\mathcal{U}D)^{\mathcal{I}(t)} & = & \{i \in \Delta^{\mathcal{I}} \mid \exists v. v > t \wedge D^{\mathcal{I}(v)}(i) \wedge \\ & & \quad \forall w. (t < w < v) \rightarrow C^{\mathcal{I}(w)}(i)\} \\ (C\mathcal{S}D)^{\mathcal{I}(t)} & = & \{i \in \Delta^{\mathcal{I}} \mid \exists v. v < t \wedge D^{\mathcal{I}(v)}(i) \wedge \\ & & \quad \forall w. (v < w < t) \rightarrow C^{\mathcal{I}(w)}(i)\} \end{array}$$

An interpretation $\mathcal{M}$ over a temporal structure $\mathcal{T} = (\mathcal{P}, <)$ satisfies a terminological axiom $C \sqsubseteq D$ if $C^{\mathcal{I}(t)} \subseteq D^{\mathcal{I}(t)}$ for every $t \in \mathcal{P}$. A knowledge base Σ is *satisfiable* in the temporal structure $\mathcal{T}$ if there is a temporal interpretation $\mathcal{M}$ over $\mathcal{T}$ which satisfies every axiom in Σ; in this case $\mathcal{M}$ is called a *model* over $\mathcal{T}$ of Σ. Σ *logically implies* an axiom $C \sqsubseteq D$ in the temporal structure $\mathcal{T}$ (written $\Sigma \models C \sqsubseteq D$) if $C \sqsubseteq D$ is satisfied by every model over $\mathcal{T}$ of Σ. In this later case, the concept C is said to be *subsumed* by the concept D in the knowledge base Σ and the temporal structure $\mathcal{T}$.

As an example of a concept using temporal operators, consider the definition of a mortal. The class of mortals denotes all the individuals which are currently living beings and live in some place, and will maintain this essence until they will stop to be living beings forever:

$$\begin{array}{ll} \mathtt{Mortal} \doteq & \mathtt{LivingBeing} \sqcap \\ & \forall \mathtt{LIVES\text{-}IN.Place} \sqcap \\ & (\mathtt{LivingBeing}\,\mathcal{U}\ \Box^{+}\neg\mathtt{LivingBeing}) \end{array}$$

The tense-logical extension of $\mathcal{ALCQIT}$ has been inspired by the works of [10, 12]. Reasoning in $\mathcal{ALCQIT}$ and in $\mathcal{ALCQIT}_{reg}$ (i.e., deciding knowledge base and concept satisfiability, deciding concept subsumption, and deciding logical implication) over a linear, unbounded, and discrete temporal structure (like the natural numbers) is decidable since it can be reduced to the decidable language $\mathcal{CIQ}_{\mathcal{US}}$ [12, 11], but its computational complexity is still unknown. In the special case of $\mathcal{ALCT}$ with empty Σ, over a linear, unbounded, and discrete temporal structure, reasoning is PSPACE-complete [10].

5. Temporal Description Logics

In the Description Logic literature, several approaches for representing and reasoning with time dependent concepts have been proposed. These temporal extensions differ from each others in different ways.

- They differ on the ontology of time, whether they adopt a point-based or an interval-based notion of time. In our case study, a point-based ontology was adopted. Interval-based temporal description logics are usually obtained from the combination with some restriction of the interval temporal modal logic $\mathcal{HS}$ [8], which is undecidable in its full power; see, for example, [1].

- They differ on the way of adding the temporal dimension, i.e., whether an explicit notion of time is adopted in which temporal operators are used to build new formulæ, or temporal information is only implicit in the language by embeddding a *state-change* based language – e.g., by resorting to a STRIPS-like style of representation – to represent sequences of events; see, for example, [4].

- In the case of an explicit representation of time, there is a further distinction between an *external* and an *internal* point of view; this distinction has been introduced by Finger and Gabbay [6].

  - In the *external* method the very same individual can have different "snapshots" in different moments of time that describe the various states of the individual at these times. In this case, a temporal logic can be seen in a modular way: while an atemporal part of the language describes the "static" aspects, the temporal part relates the different snapshots describing in such a way the "dynamic" aspects. The temporal language presented in this paper is an example of external method.
  - In the *internal* method the different states of an individual are seen as different individual components: an individual is a collection of temporal "parts" each one holding at a particular moment; see, for example, [7].

A survey analyizing the whole spectrum of approaches used within the temporal Description Logics area can be found in [2].

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